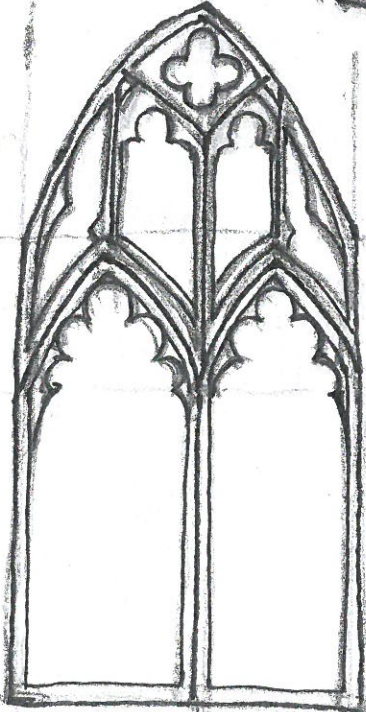
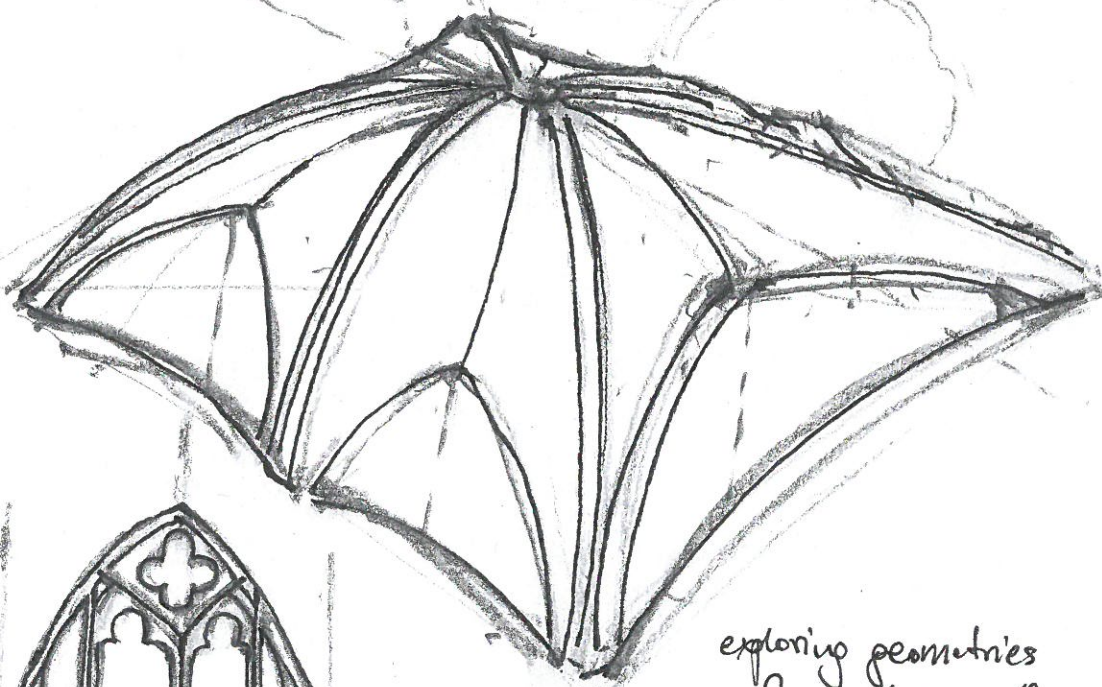
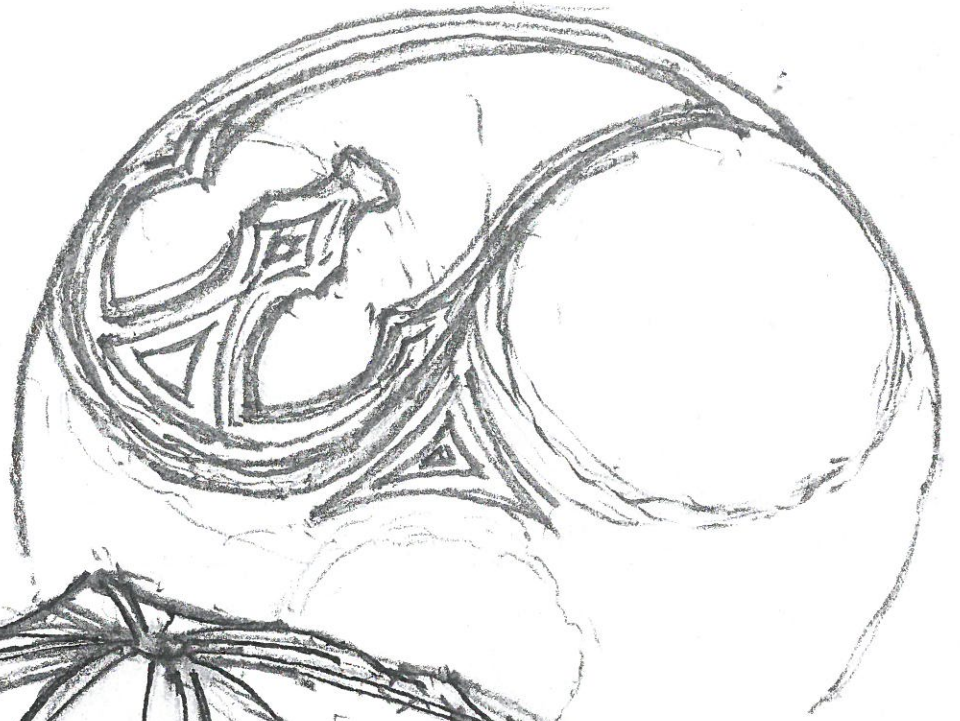
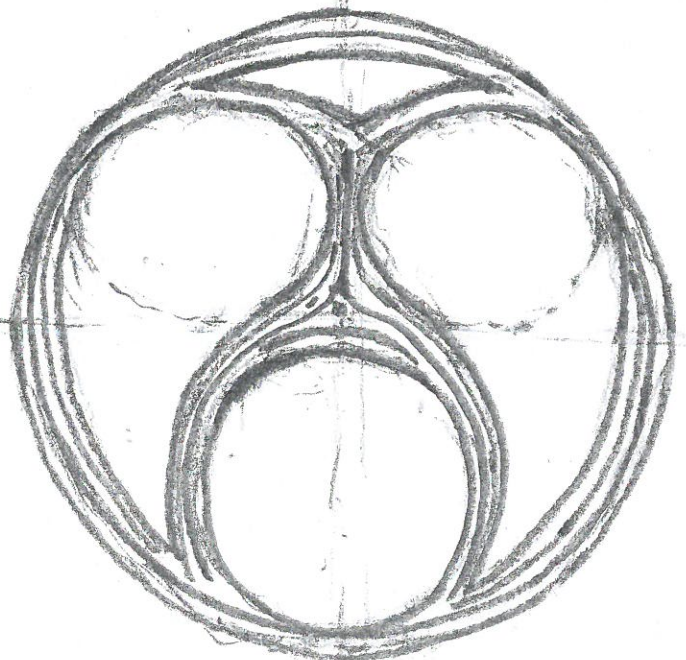
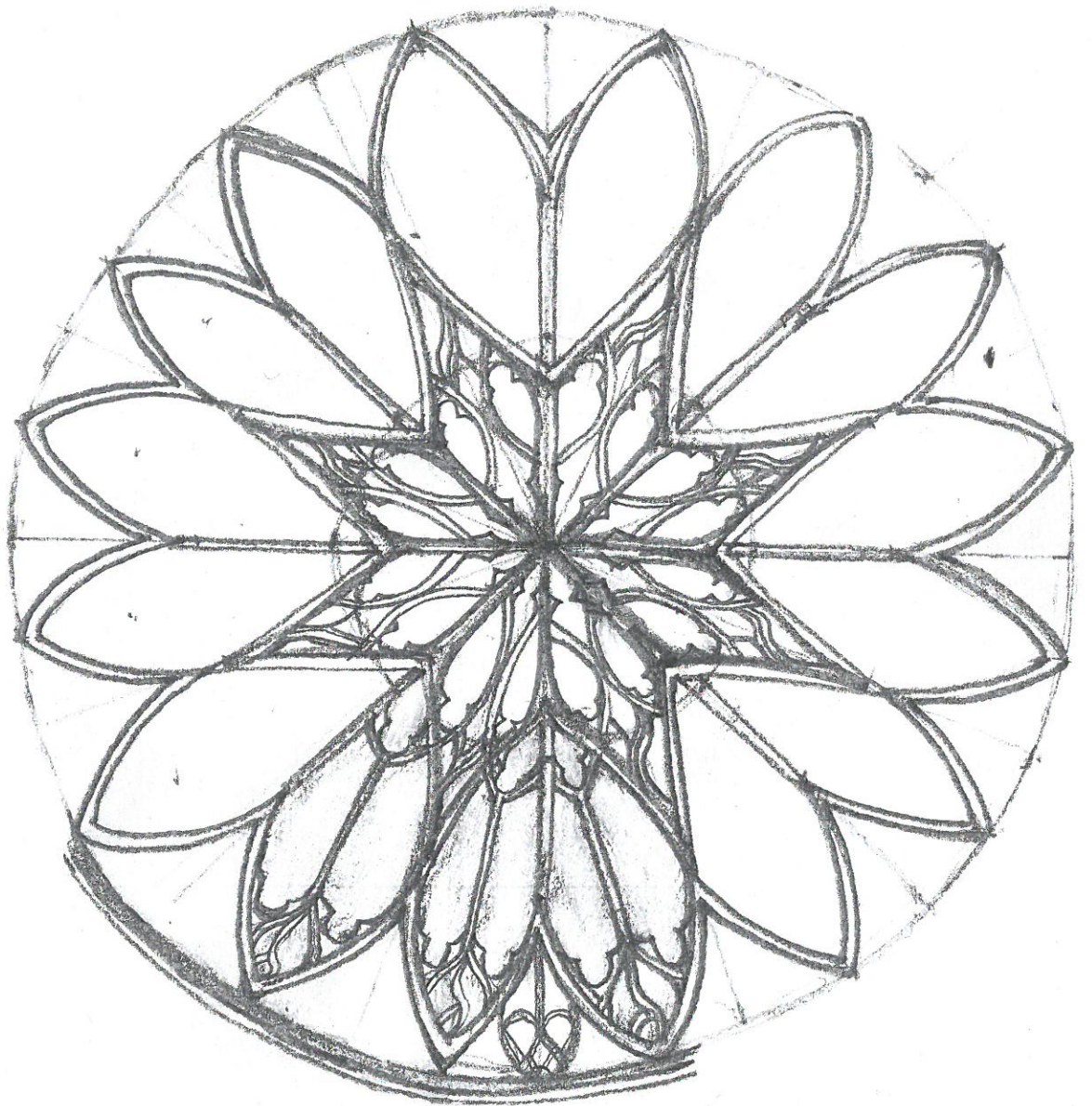


• Amiens Cathedral Rose Window



exploring geometries
for object shapes/forms





Symmetries

$$v' = Av + h$$

point / vector

• Rotation (anticlockwise)

$$A = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} \text{ and } h = 0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

centred in O of an angle φ

$\varphi = 0 \rightarrow$ identity

$\varphi = \pi \rightarrow$ symmetry through O

$\varphi \in [0, 2\pi]$ often $2\pi/n$

• Symmetry: Axis

$$A = \begin{bmatrix} \cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi \end{bmatrix} \text{ and } h = 0$$

$\varphi = 0 \rightarrow$ x axis

$\varphi = \frac{\pi}{2} \rightarrow$ y axis

symmetry axis through O

$\varphi \in [0, \pi]$

• Rotation + Scaling

$$A = \begin{bmatrix} \lambda \cos \varphi & -\lambda \sin \varphi \\ \lambda \sin \varphi & \lambda \cos \varphi \end{bmatrix} \text{ and } h = 0 \quad \lambda > 0$$

- cyclic symmetry - no axis but centre of symmetry
- dihedral symmetry: at least one symmetry axis
- more than one symmetry axis - has a centre of symmetry (the intersection of the axes)

C_n - cyclic symmetry only

D_n - having n symmetry axes (also cyclic)

C_n - n rotations of motif around the centre

D_n - can also be generated by n reflections through O

- to find the product $t_1 \circ t_2$: t_1 in row, t_2 in column (down)
- i always appear first

C_1	i
i	i

- transformed in itself by identity

D_1	i	S_1
i	i	S_1
S_1	S_1	i

- transformed in itself & by a reflection S_1 : with symmetry axis

C_2	i	\mathcal{R}_π
i	i	\mathcal{R}_π
\mathcal{R}_π	\mathcal{R}_π	i

- transformed in itself & rotation \mathcal{R}_π (with $\varphi = \pi$)

D_2	i	S_1	S_2	\mathcal{R}_π
i	i	S_1	S_2	\mathcal{R}_π
S_1	S_1	i	\mathcal{R}_π	S_2
S_2	S_2	\mathcal{R}_π	i	S_1
\mathcal{R}_π	\mathcal{R}_π	S_2	S_1	i

- transformed in itself & rotation \mathcal{R}_π ($\varphi = \pi$) & two reflections (S_1, S_2)

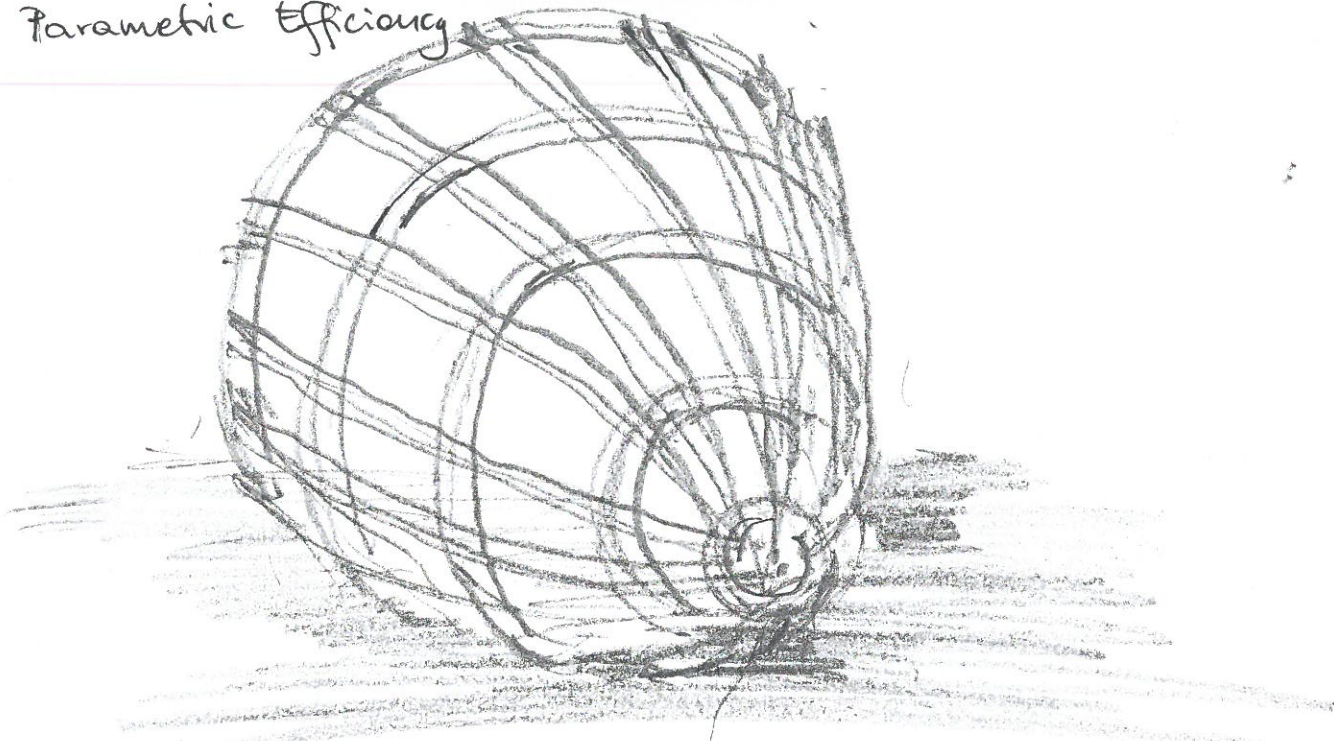
C_3	i	$\mathcal{R}_{\frac{2\pi}{3}}$	$\mathcal{R}_{\frac{4\pi}{3}}$
i	i	$\mathcal{R}_{\frac{2\pi}{3}}$	$\mathcal{R}_{\frac{4\pi}{3}}$
$\mathcal{R}_{\frac{2\pi}{3}}$	$\mathcal{R}_{\frac{4\pi}{3}}$	$\mathcal{R}_{\frac{4\pi}{3}}$	i
$\mathcal{R}_{\frac{4\pi}{3}}$	$\mathcal{R}_{\frac{2\pi}{3}}$	i	$\mathcal{R}_{\frac{2\pi}{3}}$

- transformed in itself & rotations \mathcal{R}_φ ($\varphi = \frac{2\pi}{3}, \varphi = \frac{4\pi}{3}$)

D_3	i	S_1	S_2	S_3	$\mathcal{R}_{\frac{2\pi}{3}}$	$\mathcal{R}_{\frac{4\pi}{3}}$
i	i	S_1	S_2	S_3	$\mathcal{R}_{\frac{2\pi}{3}}$	$\mathcal{R}_{\frac{4\pi}{3}}$
S_1	S_1	i	$\mathcal{R}_{\frac{2\pi}{3}}$	$\mathcal{R}_{\frac{4\pi}{3}}$	S_3	S_2
S_2	S_2	$\mathcal{R}_{\frac{2\pi}{3}}$	i	$\mathcal{R}_{\frac{4\pi}{3}}$	S_1	S_3
S_3	S_3	$\mathcal{R}_{\frac{4\pi}{3}}$	$\mathcal{R}_{\frac{2\pi}{3}}$	i	S_2	S_1
$\mathcal{R}_{\frac{2\pi}{3}}$	$\mathcal{R}_{\frac{4\pi}{3}}$	S_2	S_3	S_1	$\mathcal{R}_{\frac{4\pi}{3}}$	i
$\mathcal{R}_{\frac{4\pi}{3}}$	$\mathcal{R}_{\frac{2\pi}{3}}$	S_3	S_1	S_2	i	$\mathcal{R}_{\frac{2\pi}{3}}$

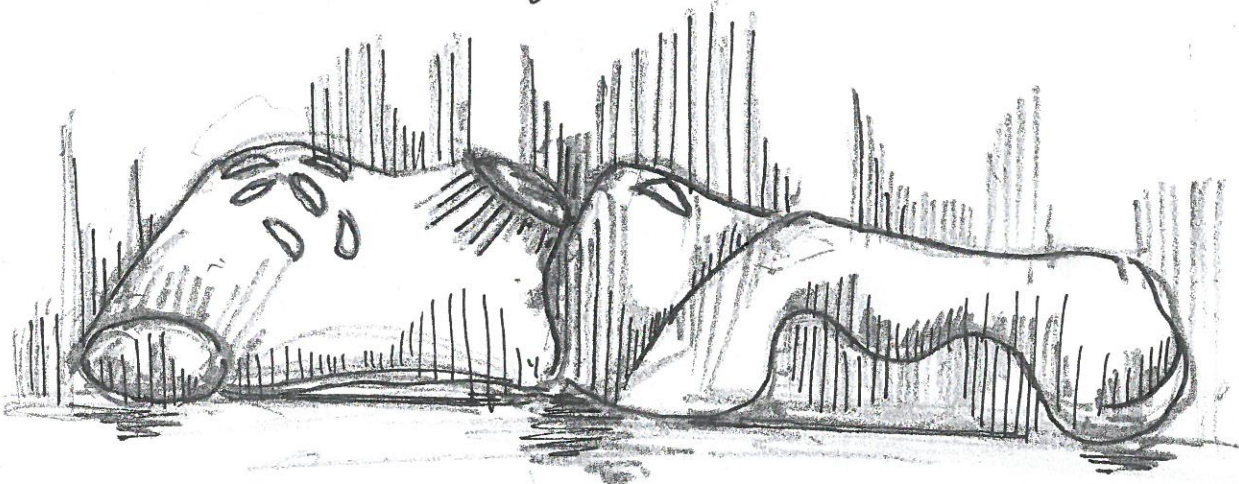
transformed in itself, by rotations $\varphi = \frac{2\pi}{3}, \varphi = \frac{4\pi}{3}$ and by reflections S_1, S_2, S_3 (axis)
 D_3 includes all previous tables

Parametric Efficiency



Comparison between traditional & new ways of construction. Scripting & parametric design could be a means to reducing carbon footprint & fuel material consumption.

Sustainable & Material Efficient Structure



Architecture in the Digital Age

- Mathematical research about rules for elements belonging to different architectural styles / eras

B.12

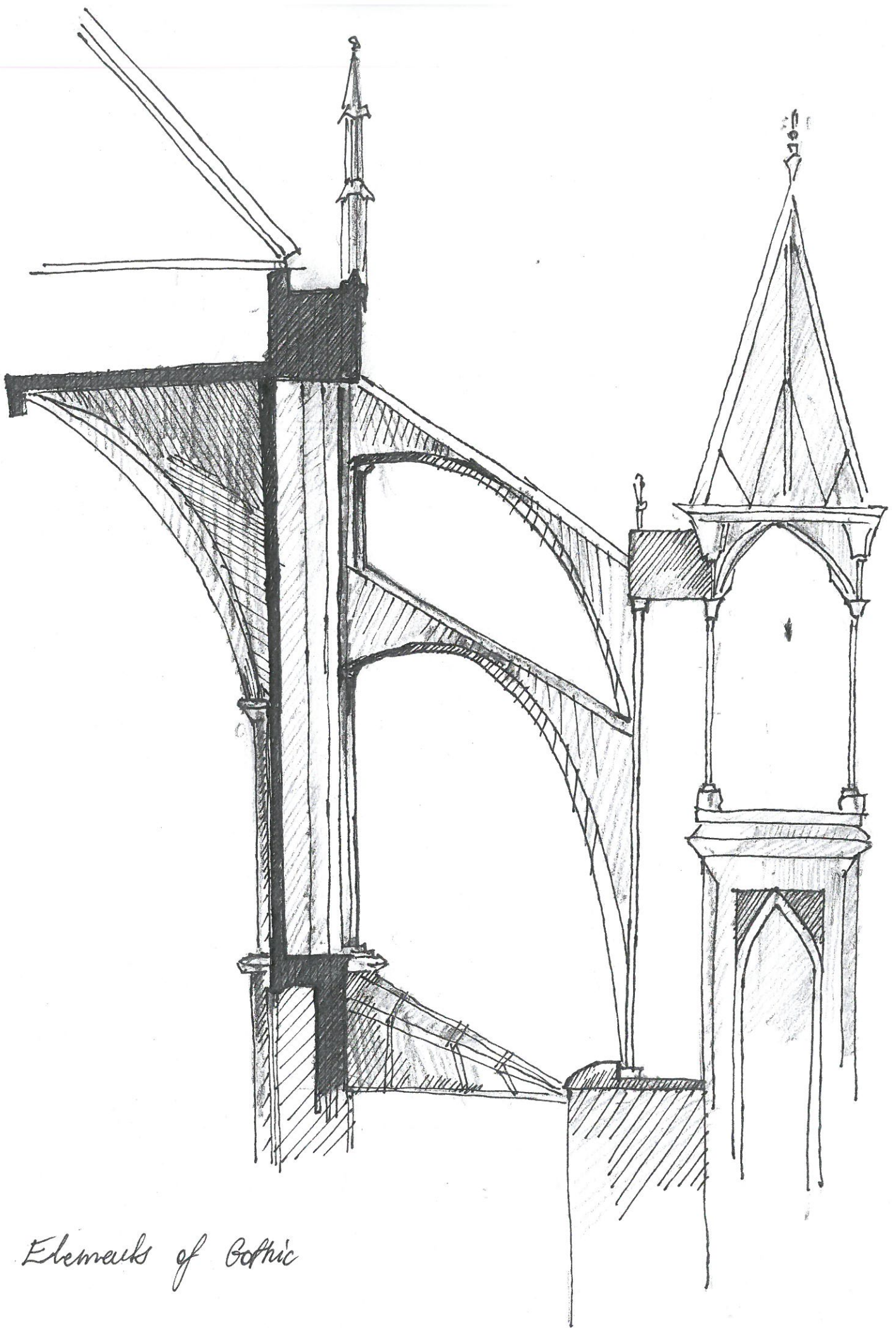
?

↓ Gothic

(Gaudi's Sagrada)

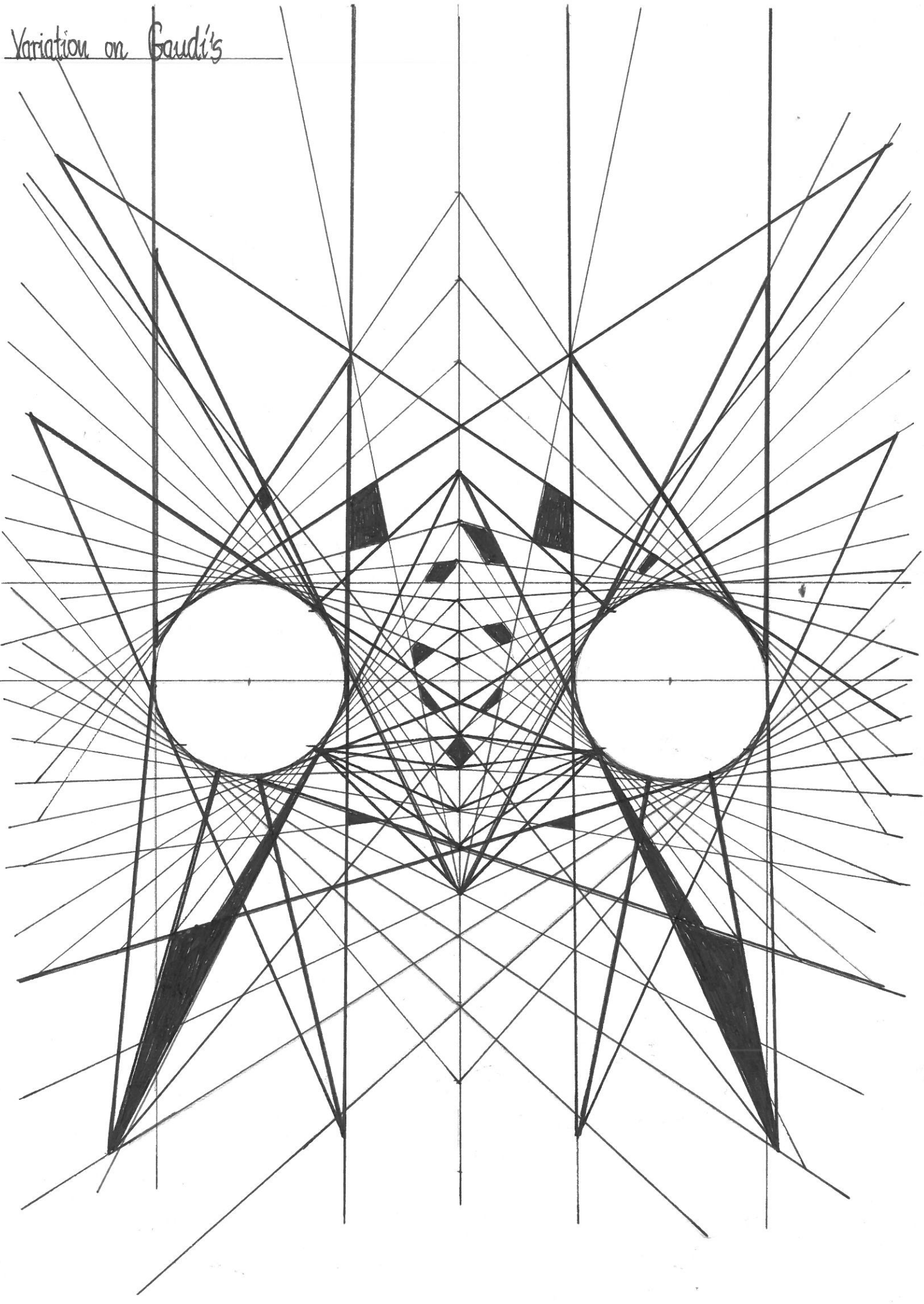
↓ Constantinian

↓ Classical

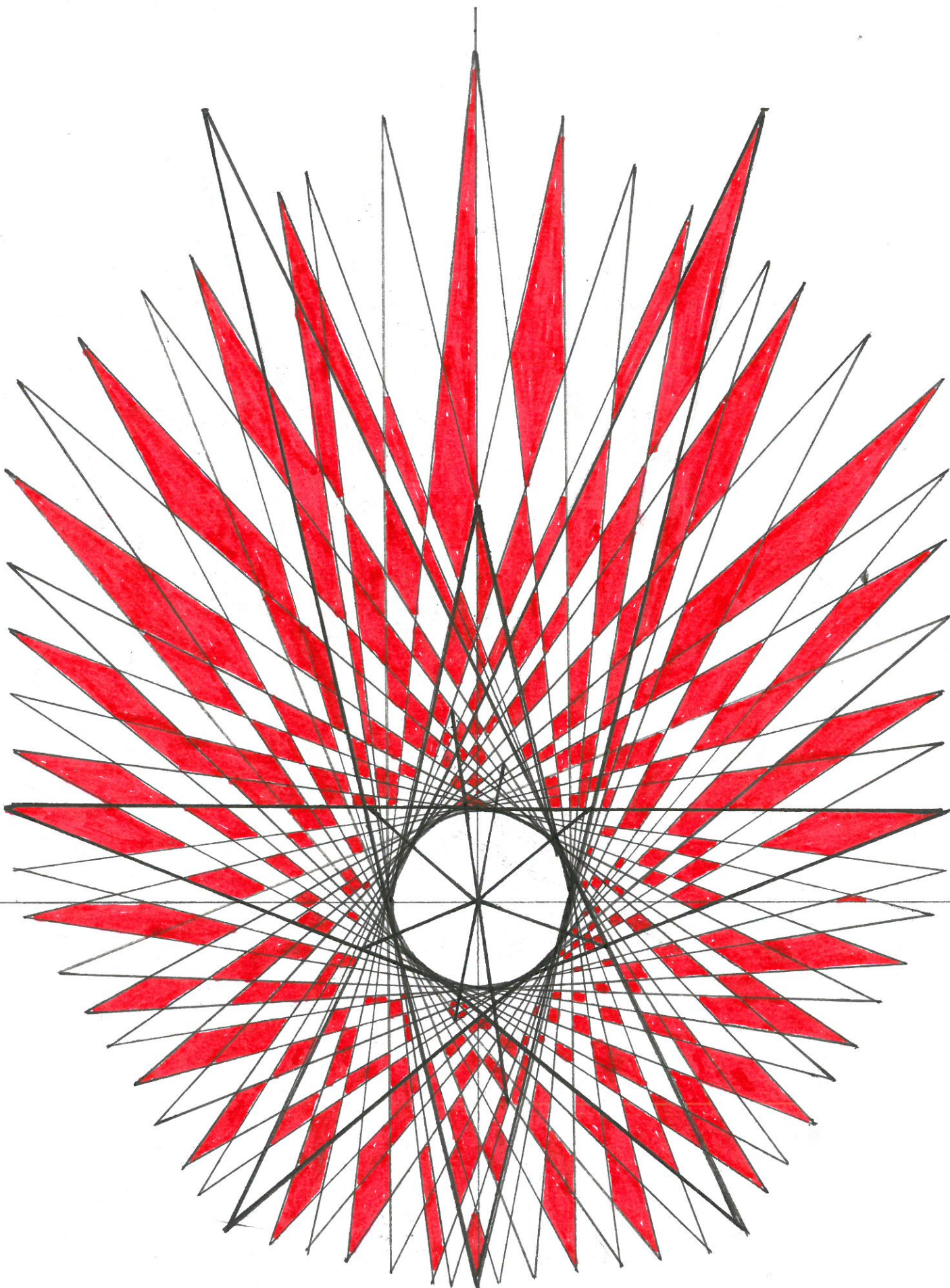


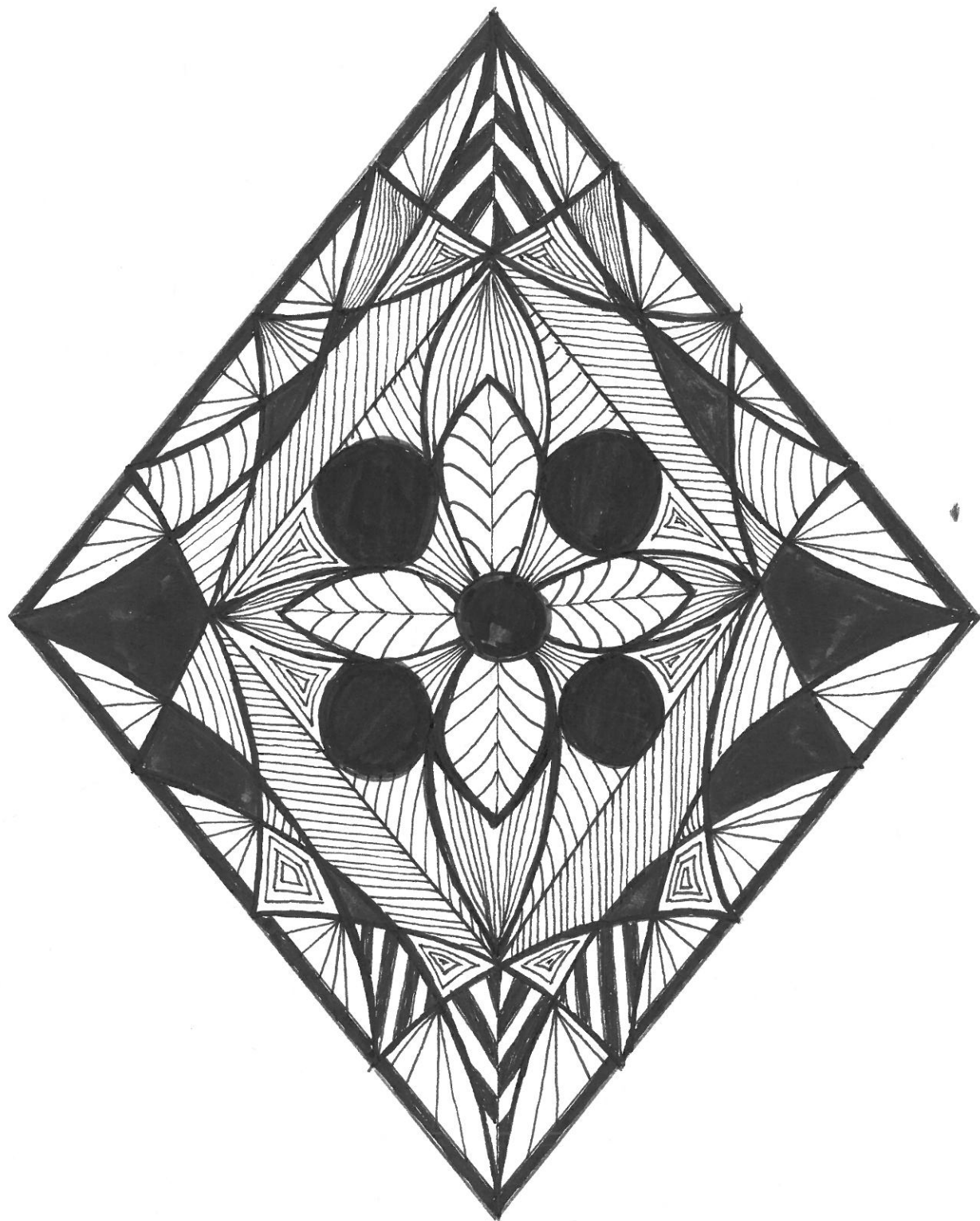
Elements of Gothic

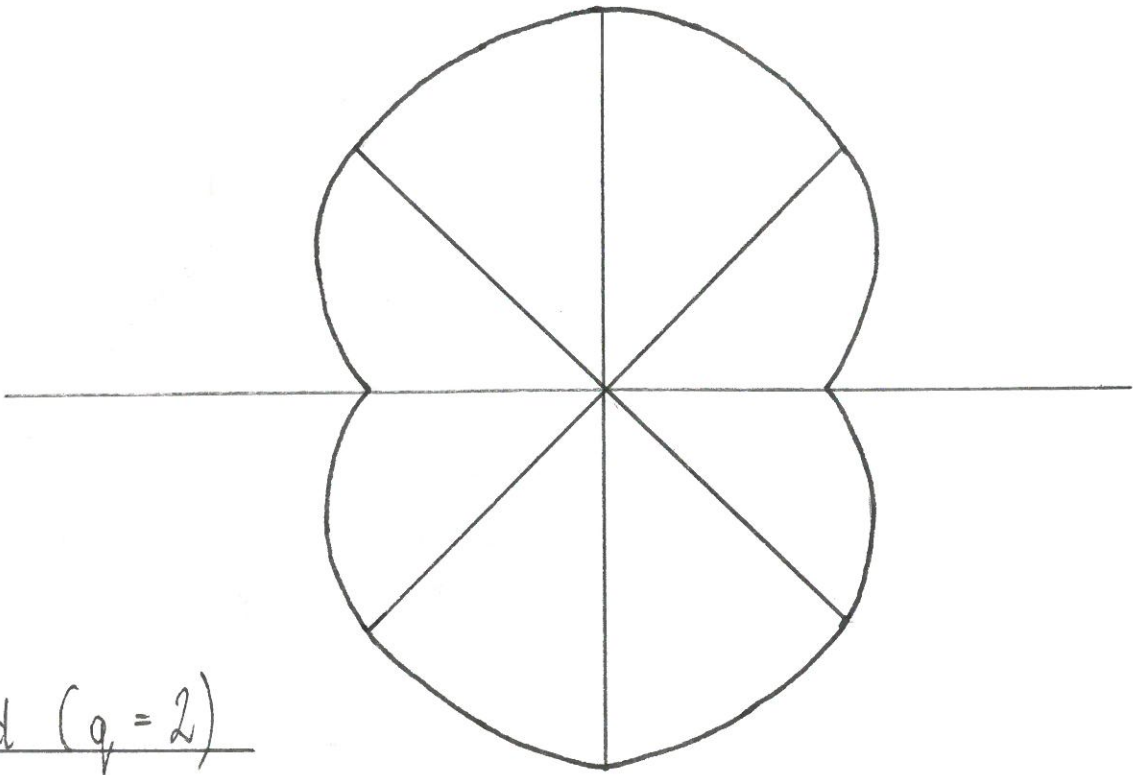
Variation on Gaudi's



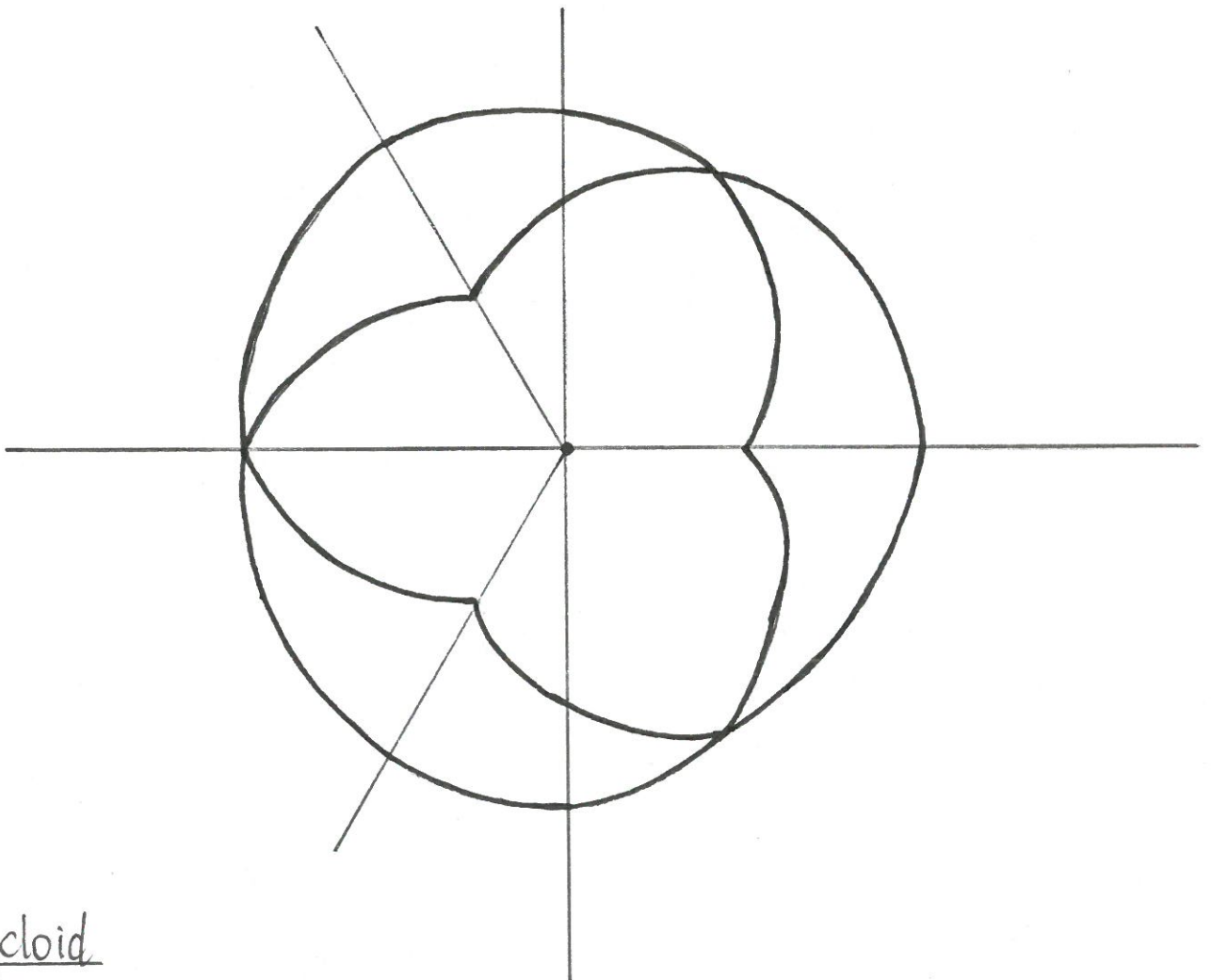
Symmetries





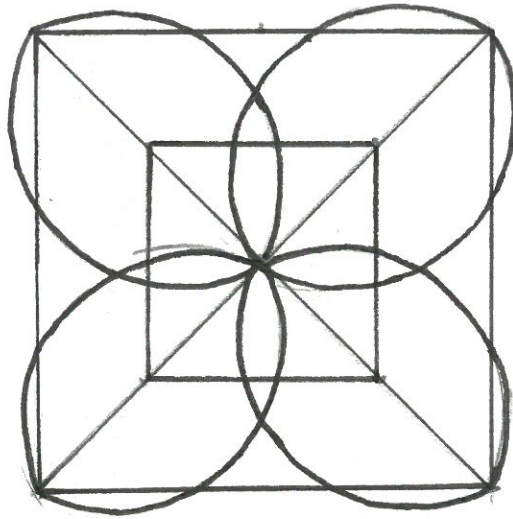


Nephroid ($q = 2$)

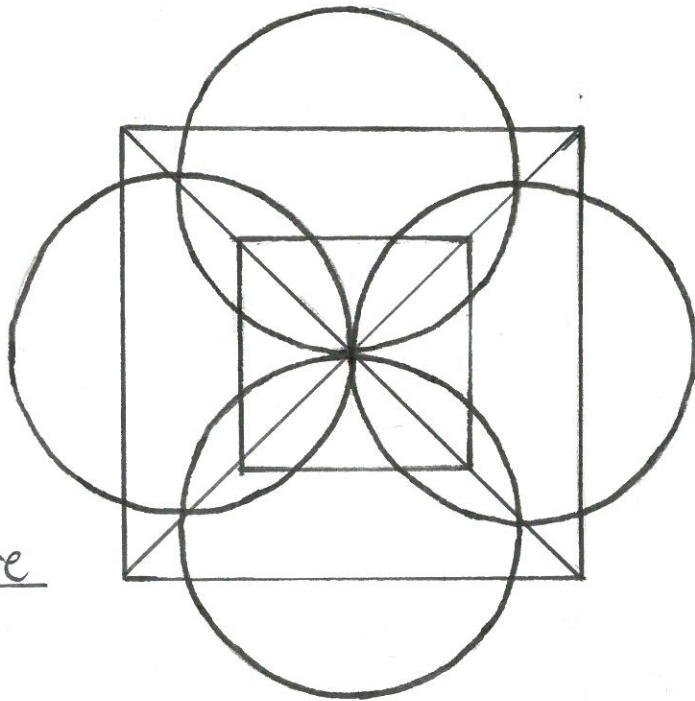


Hypocycloid

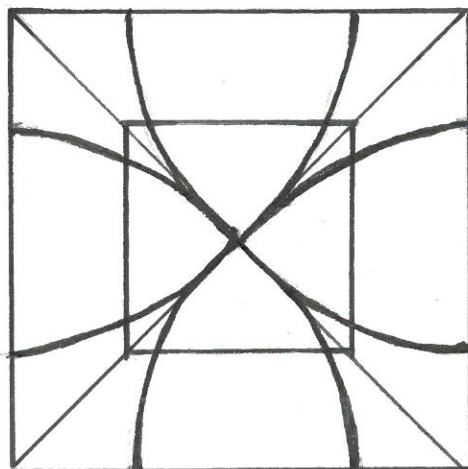
Sacred Cut



Cut: two squares



Rotated circles / centre



Arcs equal to $d/2$